

# Quantum cosmology at the turn of Millennium

A.O. Barvinsky<sup>†</sup>

*Theory Department, Lebedev Physics Institute and Lebedev Research Center in Physics, Leninsky Prospekt 53, Moscow 117924, Russia*

## Abstract

A brief review of the modern state of quantum cosmology is presented as a theory of quantum initial conditions for inflationary scenario. The no-boundary and tunneling states of the Universe are discussed as a possible source of probability peaks in the distribution of initial data for inflation. It is emphasized that in the tree-level approximation the existence of such peaks is in irreconcilable contradiction with the slow roll regime – the difficulty that is likely to be solved only on account of quantum gravitational effects. The low-energy (typically GUT scale) mechanism of quantum origin of the inflationary Universe with observationally justified parameters is presented for closed and open inflation models with a strong non-minimal coupling.

<sup>†</sup>Talk given at the IXth Marcel Grossmann Meeting, Rome, July, 2000

## 1. Introduction

This is a generally recognized fact that in the last two decades the cosmological theory was dominated by the discovery and rapid progress of the inflation paradigm that scored explaining the well known paradoxes of the standard big bang scenario. Interestingly, the beginning of inflation theory was marked in early eighties by the revival of interest in quantum cosmology. Before that it was considered merely as a toy model testing ground for quantum gravity [1]. However, practically simultaneously with the invention of the inflation scenario several suggestions were put forward for special quantum states of the Universe [2, 3, 4, 5, 6, 7] that could serve as a source of this scenario. Quantum cosmology became the theory of quantum initial conditions in inflationary Universe.

Such application of this theory was from the start marred by a number of difficulties of both conceptual and technical nature. To begin with, a great controversy broke out regarding drastic difference between two major proposals for the cosmological wavefunction – the no-boundary proposal of Hartle and Hawking [2, 3] (also semiclassically implemented in [4]) and the tunneling proposal of [5]. The origin of this difference is rooted in the peculiarities of quantum gravity theory (the presence of both positive and negative frequency solutions of the Wheeler-DeWitt equation, indefiniteness of the Euclidean gravitational action, problem

of time, etc.) which are not properly understood up till now, so that the fundamentals of these proposals and the discrepancies of their predictions are still being disputed [8].

Another problem was that, even within a rather shaky foundation of the no-boundary and tunneling wavefunctions, in the semiclassical approximation they would not generate well defined and sufficiently sharp probability peaks that could serve as a source of initial conditions for inflation. This is a very general property of all semiclassical models, and it is very easy to see this. Indeed, a basic characteristics of the inflation scenario is an effective Hubble constant  $H$ , its value determining at later times all main cosmological parameters of the Universe

$$H \implies \Omega, \rho, \frac{\delta T}{T}, \dots \quad (1.1)$$

In its turn it is usually generated by the inflaton scalar field  $\varphi$ ,  $H = H(\varphi)$ , whose initial conditions are determined by a sharp probability peak in the quantum distribution for  $\phi$ . Semiclassically this distribution  $\rho(\varphi)$  is given by the exponentiated Euclidean action of the model  $I(\varphi)$  – the Hamilton-Jacobi function in the classically forbidden domain

$$\rho(\varphi) \sim e^{\pm I(\varphi)}. \quad (1.2)$$

In the slow-roll regime the inflaton momentum is very small, which is both true for the Lorentzian and Euclidean domains (classically allowed and forbidden regions related by analytic continuation). Therefore the derivative  $dI(\varphi)/d\varphi$  is very small, the graph of  $I(\phi)$  is very flat, and it cannot generate any peak-like behaviour for the corresponding distribution function. Therefore, slow-roll nature of inflation is always in irreconcilable contradiction with the demand of the *semiclassical* probability peaks in the wave function.

Finally, in recent years another major objection arose against the issue of initial conditions for inflation in cosmology, in general, and in quantum cosmology, in particular. With the invention of self-reproducing inflation [9, 10], it was understood that the anthropic principle starts playing an important role [11, 12]. So provided, the self-reproducing eternal inflation regime is achieved, the total probability of observing some value of the effective Hubble constant equals the fundamental quantum probability  $P(H)$  weighted by the anthropic probability  $P_{\text{anthropic}}(H)$  – the probability of the existence of the observer. The latter is obviously proportional to the volume of inflating Universe and, therefore, exponentially depending on time,  $P_{\text{anthropic}}(H) \sim \exp(3Ht)$ . Therefore, very quickly any probability peak of  $P(H)$  gets wiped out by the anthropic factor in  $P_{\text{total}}(H) \sim P(H) \exp(3Ht)$ , unless the peak  $P(H)$  has a stronger fall off behaviour in  $H$  than the exponential one.

In this paper we give a brief overview of the present state of quantum cosmology and advocate that despite the intrinsic difficulties and objections of the above type this theory remains a viable scheme of description for the very early quantum Universe. In particular, we shall try to show that its imprint on the observable large scale structure can be used as a testing ground of fundamental quantum gravity theory in the range of energies where the semiclassical expansion can be trusted. We would like to emphasize here that the role of quantum initial conditions in cosmology should not be underestimated. One of the reasons is that these conditions determine the energy scale of the cosmological evolution (encoded in the characteristic value of the Hubble constant) as compared to both the Planckian scale, where semiclassical methods break, and the self-reproduction scale of entering the eternal inflation.

Note that the argumentation against the issue of initial conditions in cosmology usually starts with the assumption of the Planckian energy scale at the onset of inflation, resulting in the self-reproduction conditions for eternal inflation. However, such a starting point cannot be regarded reliable because of the absence of non-perturbative methods at high energies and the absence of fully consistent quantum gravity theory at this energy scale. Within the conventional perturbation framework the predictions can be justified only when the entire evolution of the system stays in the low-energy domain. If this evolution corresponds to the conventional inflation scenario followed by the standard big-bang model cosmology evolving down to lower scales, then the criterion of our semiclassical predictions boils down to verification of the initial energy scale being much below the Planckian one. This is the point where the issue of initial conditions becomes crucially important – this low initial scale should not be imposed by hands, but rather be derived from general assumptions on the cosmological quantum state. One of the goals of this review is to demonstrate that such a possibility occurs in the cosmological model with strong non-minimal curvature coupling of the inflaton – the model for which the quantum origin of the Universe turns out to be the low-energy (typically GUT) phenomenon generating the present day observable cosmological parameters.

The paper is organized as follows. We start with a brief overview of the state of art in quantum cosmology in the year 2000, discuss the two fundamental proposals for the cosmological quantum state – the no-boundary and tunneling wavefunctions, their implications in context of the closed and open cosmology and then show how these states can lead to the low energy phenomenon of the quantum origin of the inflationary Universe.

## 2. Quantum cosmology 2000 – state of art

State of art in present day quantum cosmology consists of the set of rules, rendering this field a physically complete and consistent theory, and the scope of approximation methods that allow one to solve quantum cosmological problems in concrete setting. Among these main ingredients one can single out three basic subjects most important for applications: i) Wheeler-DeWitt equation and the path integral; ii) semiclassical approximation and beyond; iii) collective variables – homogeneous minisuperspace modes and cosmological perturbations.

### 2.1. Wheeler-DeWitt equation and path integral

The formalism of quantum cosmology is based on the system of Wheeler-DeWitt equations – the quantum Dirac first-class constraints as equations selecting the quantum state of the gravitational and matter fields  $\Psi[g_{ab}(\mathbf{x}), \phi(\mathbf{x})]$  in the functional coordinate representation of canonical commutation relations for the operators of 3-metric  $g_{ab}(\mathbf{x})$ , matter fields  $\phi(\mathbf{x})$  and their conjugated momenta  $p^{ab}(\mathbf{x}) = \delta/i\delta g_{ab}(\mathbf{x})$ ,  $p(\mathbf{x}) = \delta/i\delta\phi(\mathbf{x})$ . These equations look like

$$\begin{aligned}\hat{H}_\perp(\mathbf{x})\Psi[g_{ab}(\mathbf{x}), \varphi(\mathbf{x})] &= 0, \\ \hat{H}_a(\mathbf{x})\Psi[g_{ab}(\mathbf{x}), \varphi(\mathbf{x})] &= 0.\end{aligned}\tag{2.1}$$

Here  $\hat{H}_\perp(\mathbf{x})$  and  $\hat{H}_a(\mathbf{x})$  are the operators of the superamiltonian and supermomentum constraints – the functional variation operators of the second and first order correspondingly whose actual form will not be very important for us.

From the viewpoint of local quantum field theory such operators in the functional Schrodinger representation are not well-defined because of poorly defined equal time products of local operators, but formally this problem is overcome by writing down a formal path integral solution to these equations – which all the same does not control the equal-time commutators

$$\Psi[g_{ab}(\mathbf{x}), \varphi(\mathbf{x})] = \int Dg_{\mu\nu}(x) D\phi(x) \exp\left(\frac{i}{\hbar} S[g_{\mu\nu}(x), \phi(x)]\right) \Big|_{\text{gauge fixing}}, \quad (2.2)$$

$$g_{\mu\nu}, \phi \Big|_\Sigma = g_{ab}(\mathbf{x}), \varphi(\mathbf{x}). \quad (2.3)$$

Here the integration runs over 4-metrics and matter fields in spacetime domain subject to boundary values – 3-metric  $g_{ab}(\mathbf{x})$  and matter field  $\varphi(\mathbf{x})$  – arguments of the wavefunction defined on this spacelike boundary.

This integral in the naive form lacking the gauge fixation was first proposed by H. Leutwyler [13] and later derived in [14, 15] with a proper account of the full Feynman-DeWitt-Faddeev-Popov gauge fixing procedure and boundary conditions on integration variables (for later and more detailed formulation see [16, 17]). The Euclidean version of this path integral representation for the solution of Wheeler-DeWitt equations was then used by Hartle and Hawking in the formulation of the no-boundary prescription for the cosmological wavefunction [2, 3].

## 2.2. Semiclassical approximation and beyond

Usefulness of the path integral consists, as is well known, in the possibility of developing a regular semiclassical expansion. In quantum cosmology, however, the knowledge of the solution to Wheeler-DeWitt equations – exact or within any approximation scheme – does not guarantee the exhaustive solution of the physical problem.

One of the reasons is that, in contrast with usual quantum field theory, the basic Wheeler-DeWitt equation is not evolutionary – unlike the Schrodinger equation it is hyperbolic rather than parabolic in its variables, and, moreover, it is not just one equation, but the whole infinite system of equations, the consistency of which is guaranteed by the closure of the algebra of operators in the left-hand side of eqs.(2.1). For this reason, the formalism of quantum cosmology is devoid of a unique time variable labeling the evolution, which is nothing but the manifestation of the diffeomorphism invariance of the theory. This simple property at the classical level results in disastrous complications at the quantum level – sometimes called the problem of time, the remarkable review of its status given by K. Kuchar [18]. This problem has a number manifestations which, however, altogether originate from the problem of interpreting the cosmological wavefunction – the solution of eqs.(2.1).

In contrast with the evolutionary Schrodinger problem of a conventional QFT, this interpretation is far from being obvious. To begin with, the Wheeler-DeWitt equations in view of their hyperbolic nature imply as a conserved object the quantity which is not positive definite – a direct analogue of the situation with the Klein-Gordon equation in first-quantized theory. Therefore, this conserved quantity – inner product in the space of solutions of the Wheeler-DeWitt equations – cannot be used for constructing probability amplitudes. Another side

of this problem is the presence of solutions of both positive and negative frequencies contributing with opposite signs to the probabilistic quantities. By and large, this problem has not yet been solved, so we present here only the existing preliminary steps in its partial resolution. Interestingly, those steps were undertaken simultaneously with the construction of semiclassical expansion for the cosmological wavefunction, which is very often interpreted as the fact that time (and associated with it conserved probability) in quantum gravity is not a fundamental concept, but rather is the notion which arises only in semiclassical approximation. We believe, though, that such a widespread opinion is erroneous – lack of our understanding should not be hidden by smearing out the fundamental and primary notions of physics.

The first serious attempt to introduce time, probability, etc. in quantum cosmology originated from the semiclassical approximation for  $\Psi[g_{ab}(\mathbf{x}), \phi(\mathbf{x})]$  in the sector of the gravitational field  $g_{ab}(\mathbf{x})$ . This was first done, at the level of minisuperspace model, by DeWitt in his pioneering paper on canonical quantum gravity [1] and then rederived for generic gravitational system by Rubakov and Lapchinsky in [19] and also intensively discussed by Banks [20]. With the wavefunction  $\Psi[g_{ab}(\mathbf{x}), \phi(\mathbf{x})]$  rewritten as

$$\Psi[g_{ab}(\mathbf{x}), \phi(\mathbf{x})] = \exp\left(\frac{i}{\hbar} S[g_{ab}(\mathbf{x})]\right) \Psi_m[g_{ab}(\mathbf{x}), \phi(\mathbf{x})], \quad (2.4)$$

where  $S[g_{ab}(\mathbf{x})]$  is the Einstein-Hamilton-Jacobi function of the pure gravitational field in vacuum, the function  $\Psi_m[g_{ab}(\mathbf{x}), \phi(\mathbf{x})]$  starts playing the role of the quantum state of quantized matter fields in the external classical gravitational background. Such an interpretation is justified by the fact that, when this function is restricted to the solution of classical Einstein equations in vacuum,  $g_{ab}(\mathbf{x}) = g_{ab}^{\text{class}}(t, \mathbf{x})$ , then it becomes the explicit function of time variable,  $\Psi_m(t)[\phi(\mathbf{x})] \equiv \Psi_m[g_{ab}^{\text{class}}(t, \mathbf{x}), \phi(\mathbf{x})]$ . Moreover, on account of the Wheeler-DeWitt equations it satisfies in the lowest order approximation in  $1/m_P^2$  – the inverse of the Planck mass squared – the Schrodinger equation with the quantum Hamiltonian of quantized matter fields in *external classical gravitational field without sources*. In operator notations,  $|\Psi_m(t)\rangle = \Psi_m(t)[\phi(\mathbf{x})]$ ,

$$i\hbar \frac{\partial}{\partial t} |\Psi_m(t)\rangle = \int d^3x (N^\perp \hat{H}_\perp + N^a \hat{H}_a) |\Psi_m(t)\rangle, \quad (2.5)$$

where the operator Hamiltonian is explicitly written as a linear combination of matter superhamiltonian and supermomenta with the background lapse and shift functions as coefficients.

Thus, when the semiclassical approximation for the gravitational field is reliable, the time variable can be introduced into the formalism of quantum cosmology by means of the classical gravitational background that i) neither takes into account the back reaction of quantized matter nor ii) has its own quantum fluctuations. The discussion of such a way of introducing time in cosmology can be found in [19, 16, 21, 22]. Majority of results in modern cosmology have been obtained within this interpretation framework – with time introduced via the classical background. An obvious limitation of this framework is that the quantum properties of the gravitational background are inaccessible and it is incapable of accounting for quantum back reaction properties – so, in essence, this is not quantum and cosmology but rather quantum field theory in curved spacetime. The origin of this difficulty is obvious – the quantum effects of the gravitational and matter fields are not treated on equal footing:

the Shrodinger equation of the above type takes into account quantum effects of matter exactly (to all powers in  $\hbar$ ), but disregards all inverse powers of the Planck mass squared  $1/m_P^2$ . The way around this difficulty is to develop a regular semiclassical loop expansion in  $\hbar$  without distinguishing those powers of Planck's constant that arise from  $1/m_P^2$  and those from the quantum loops of matter fields, and, simultaneously not to loose time and probability interpretation inherent in the Schrodinger equation above.

This program was implemented in the series of papers [23, 24, 16, 25] in the lowest non-trivial order of  $\hbar$ -expansion for quantum states having the form of a single semiclassical wave packet

$$\Psi[g_{ab}(\mathbf{x}), \varphi(\mathbf{x})] = P[g_{ab}(\mathbf{x}), \varphi(\mathbf{x})] \exp\left(\frac{i}{\hbar} S[g_{ab}(\mathbf{x}), \varphi(\mathbf{x})]\right). \quad (2.6)$$

Here  $S[g_{ab}(\mathbf{x}), \varphi(\mathbf{x})]$  is the Hamilton-Jacobi system of the full system of interacting gravitational and matter fields, and

$$P[g_{ab}(\mathbf{x}), \varphi(\mathbf{x})] = P_{1\text{-loop}}[g_{ab}(\mathbf{x}), \varphi(\mathbf{x})] + O(\hbar) \quad (2.7)$$

is the preexponential factor whose expansion in  $\hbar$  begins with the one-loop term.

In [23, 24, 16] it was shown that the one-loop preexponential factor can be universally obtained in terms of the Hamilton-Jacobi function of the system. This factor was obtained by both solving the Wheeler-DeWitt equations in the approximation linear in  $\hbar$  [23, 24] and calculating the path integral in the gaussian approximation [17]. Both methods give the same result, thus confirming formal consistency of the theory. The expression for the conserved inner product of semiclassical quantum states was derived [23, 24, 25] (with a nontrivial measure in superspace of 3-metrics and matter fields). This inner product turned out to coincide with the usual inner product of states in the physical sector of the theory arising as a result of the Hamiltonian (or ADM) reduction to true physical degrees of freedom. Physical time arising in this reduction was shown to survive the transition to the one-loop approximation, which means that concept of time is not entirely semiclassical, but admits continuation to quantum domain. All these conclusions attained in the one-loop approximation can be generalized to higher orders of semiclassical expansion by the price of technically complicated formalism – the only principal limitation of this framework is that the starting point remains the semiclassical wave packet of the form (2.6) not mixing opposite frequencies. This is a fundamental limitation that reflects conceptual problems in quantum cosmology, that are not yet resolved.

### 2.3. Collective variables – minisuperspace and cosmological perturbations

The success of applications in a physical theory to an essential extent depends on a particular approximation scheme used. Together with general semiclassical expansion, considered above, one should use approximation schemes that simplify the configuration space of the theory leaving aside those degrees of freedom which are not very important in the problem in question. The extremal manifestation of this approach in cosmology consists in the so called minisuperspace reduction, when only a finite number of collective degrees of freedom is left – describing the spatially homogeneous cosmological model. At the quantum

level, such approximation is not completely consistent, because the zero-point fluctuations of the discarded degrees of freedom cannot be excluded by hands – they might give an important contribution to the dynamics of those collective variables that determine the main features of the model. Thus, more fruitful is the approach when all the degrees of freedom are retained, but a finite number of them are treated exactly, while the rest are considered perturbatively in the linearized approximation and higher. Such a scheme matches well with the semiclassical expansion in which quantum fluctuations of linearized modes give contributions to perturbative loop effects.

In application to cosmology, this general scheme implies a well known theory of cosmological perturbations (see, for example [27]), when the metric and matter field are decomposed into the spatially homogeneous background and inhomogeneous perturbations

$$ds^2 = -N^2(t)dt^2 + a^2(t)\gamma_{ij}dx^i dx^j + h_{\mu\nu}(x)dx^\mu dx^\nu, \quad (2.8)$$

$$\varphi(x) = \varphi(t) + \delta\varphi(x), \quad x = (t, \mathbf{x}), \quad (2.9)$$

where  $a(t)$  is the scale factor,  $N(t)$  is the lapse function and  $\gamma_{ij}$  is the homogeneous spatial metric (for closed cosmology this is a metric of the 3-sphere of unit radius). The full set of fields consists of the minisuperspace sector of spatially homogeneous variables ( $a(t)$ ,  $\varphi(t)$ ,  $N(t)$ ) and inhomogeneous fields  $f(x)$  essentially depending on spatial coordinates  $x^i = \mathbf{x}$

$$f(x) = \delta\varphi(t, \mathbf{x}), h_{\mu\nu}(t, \mathbf{x}), \chi(t, \mathbf{x}), \psi(t, \mathbf{x}), A_\mu(t, \mathbf{x}), \dots \quad (2.10)$$

The role of classically non-vanishing scalar field  $\varphi$  (or the field with non-vanishing expectation value) is usually played by inflaton that drives the quasi-exponential expansion at the inflationary stage of the evolution. On the other hand, the cosmological perturbations of metric, the inflaton field itself and other matter fields – initially quantum and later semiclassically coherent – describe the formation of the large scale structure on the Robertson-Walker cosmological background (including microwave background radiation and other matter in the observable Universe).

The description of cosmological perturbations requires the formalism that deals with its gauge invariance properties [28, 27] – not all the variables above are dynamically independent and physically significant, because part of the variables represent just purely coordinate degrees of freedom or those degrees of freedom that can be excluded in virtue of constraints in terms of physical variables. The dynamical content of the latter is basically the following. The spatially homogeneous sector of inflaton  $\varphi$ , scale factor  $a$  and lapse  $N$  gives rise to only one dynamical degree of freedom – it can be without loss of generality identified with the inflaton  $\varphi$ , while for the inhomogeneous perturbations, basically it is transverse, transverse-traceless, etc., components that form the physical sector. We shall collectively denote them by  $f^T$ .

With such a decomposition, the effective dynamics of the main collective variable in cosmology – the inflaton field  $\varphi$  is determined by the reduced density matrix obtained by tracing out the inhomogeneous fields  $f^T$ . If in the physical sector the full cosmological state is denoted by  $\Psi(\varphi, f^T)$ , then this density matrix is given by

$$\hat{\rho} \equiv \rho(\varphi, \varphi') = \int df^T \Psi(\varphi, f^T) \Psi^*(\varphi', f^T). \quad (2.11)$$

The diagonal element of this density matrix is the distribution function of the inflaton  $\varphi$ ,

$$\rho(\varphi) = \rho(\varphi, \varphi), \quad (2.12)$$

which might yield initial conditions for inflation, provided it has a peak-like behaviour for some suitable mean values of  $\varphi$ . This goes as follows.

In the chaotic inflation model the stage of inflation is generated within the slow roll approximation, when the inflaton field slowly rolls down the the potential  $V(\varphi)$  – some monotonically growing function of  $\varphi$  – which, in its turn, determines a big value of the effective Hubble constant,  $\dot{a}/a \simeq H(\phi)$ ,

$$H^2(\phi) = \frac{8\pi V(\varphi)}{3m_P^2}. \quad (2.13)$$

The initial value of  $\varphi$  actually determines all main cosmological parameters, including the duration of inflation in terms of the e-folding number  $N$  – the logarithmic expansion coefficient for the cosmological scale factor  $a$  during the inflation stage,

$$N = \int_0^{t_F} dt H \quad (2.14)$$

(with  $t = 0$  and  $t_F$  denoting the beginning and the end of inflation epoch) and the present day value of  $\Omega$  [29],

$$\Omega \simeq \frac{1}{1 \mp B \exp(-2N)}, \quad (2.15)$$

The signs  $\mp$  here are related respectively to the closed and open models, and  $B$  is the parameter incorporating the details of the reheating and radiation-to-matter transitions in the early Universe. Depending on the model for these transitions, its order of magnitude can vary from  $10^{25}$  to  $10^{50}$  (when the reheating temperature varies from the electroweak to GUT scale). In what follows we shall assume the latter as the most probable value of this parameter.

Eq. (2.15) clearly demonstrates rather stringent bounds on  $N$ . For the closed model the e-folding number should satisfy the lower bound  $N \geq \ln B/2 \simeq 60$  in order to generate the observable Universe of its present size, while for the open model  $N$  should be very close to this bound  $N \simeq 60$  in order to have the present value of  $\Omega$  *not very close to zero or one*,  $0 < \Omega < 1$ , the fact intensively discussed on the ground of the recent observational data.

In the chaotic inflation model the effective Hubble constant is generated by the potential of the inflaton scalar field and all the parameters of the inflationary epoch, including its duration in units of  $N$ , can be found as functions of the initial value of the inflaton field  $\varphi$  at the onset of inflation  $t = 0$ . If this initial condition belongs to the quantum domain then it has to be considered subject to the quantum distribution (2.11)-(2.12) following from the cosmological wavefunction. If this distribution function has a sharp probability peak at certain  $\varphi$ , then, at least within the semiclassical expansion, this value of  $\varphi$  serves as the initial condition for the inflationary dynamics.

### 3. No-boundary and tunneling wave functions

Quantum states that give initial conditions for inflation were suggested in early eighties. Basically, these are two states having in the semiclassical approximation qualitatively different behaviours. One of these states – the so called no-boundary one – was suggested by Hartle and Hawking [2, 3] in the form of the Euclidean path integral prescription. This prescription reduces to the Euclidean version of the integral (2.3) where the integration goes over Euclidean compact 4-geometries and 4-dimensional histories of matter fields "bounded" by the 3-geometry and matter field – the arguments of the cosmological wavefunction. The underlying spacetime of Euclidean signature has the topology of a 4-dimensional ball. Another quantum state – the tunneling one – was suggested as a particular semiclassical solution of the minisuperspace Wheeler-DeWitt equation in the Robertson-Walker model with the inflaton potential, generating the effective Hubble (or cosmological constant) [5, 6, 7].

The both states were analyzed in much detail for the model of minimally coupled inflaton field  $\varphi$  having a generic potential  $V(\varphi)$

$$S[g_{\mu\nu}, \varphi] = \int d^4x g^{1/2} \left( \frac{m_P^2}{16\pi} R(g_{\mu\nu}) - \frac{1}{2}(\nabla\varphi)^2 - V(\varphi) \right). \quad (3.1)$$

As semiclassical solutions of the minisuperspace Wheeler-DeWitt equation in this model, they can be written down in the slow roll approximation (when the derivatives with respect to  $\varphi$  are much smaller than the derivatives with respect to  $a$ ). They read [6, 7]

$$\Psi_{NB}(\varphi, a) = C_{NB} (a^2 H^2(\varphi) - 1)^{-1/4} \exp \left[ -\frac{1}{2} I(\varphi) \right] \cos \left[ S(a, \varphi) + \frac{\pi}{4} \right], \quad (3.2)$$

$$\Psi_T(\varphi, a) = C_T (a^2 H^2(\varphi) - 1)^{-1/4} \exp \left[ +\frac{1}{2} I(\varphi) + iS(a, \varphi) + \frac{i\pi}{4} \right] \quad (3.3)$$

and describe two types of the nucleation of the Lorentzian quasi-DeSitter spacetime (described by the Hamilton-Jacobi function  $S(\varphi, a)$ ) from the gravitational semi-instanton – the Euclidean signature hemisphere bearing the Euclidean gravitational action  $I(\varphi)/2$

$$\begin{aligned} I(\varphi) &= -\frac{\pi m_P^2}{H^2(\varphi)}, \\ S(\varphi, a) &= -\frac{\pi m_P^2}{2H^2(\varphi)} (a^2 H^2(\varphi) - 1)^{3/2}. \end{aligned} \quad (3.4)$$

The size of this hemisphere – its inverse radius – as well as the curvature of the quasi-DeSitter spacetime are determined by the effective Hubble constant (2.13)

In the tree-level approximation the quantum distributions of universes with different values of the inflaton field  $\phi$  (2.11)-(2.12) are, thus, given by two algorithms – the real amplitudes of (3.2)-(3.3):

$$\rho_{NB}(\varphi) \sim e^{-I(\varphi)} \quad (3.5)$$

for the no-boundary quantum state [2, 3, 4] and

$$\rho_T(\phi) \sim e^{I(\varphi)}, \quad (3.6)$$

for the tunneling one [5]. For the *minimally* coupled inflaton,  $I(\varphi)$  – the action of the gravitational instanton – reads, up to inflationary slow roll corrections, as

$$I(\phi) \simeq -\frac{3m_P^4}{8V(\varphi)}, \quad (3.7)$$

where  $V(\varphi)$  is the inflaton potential. From these equations it immediately follows that the no-boundary and tunneling wavefunctions lead to opposite predictions: most probable universes with a minimum of the inflaton potential in the no-boundary case and with a maximum – for the tunneling situation [6].

However, for reasons discussed in Introduction these extrema in the probability distribution cannot generate inflationary scenario. The main obstacle is the irreconcilable contradiction between the slow-roll nature of inflation and the requirement of sharp probability peaks – for slow-roll regime the  $\varphi$ -derivatives of the distribution function are very small, because by order of magnitude they coincide with the momentum conjugated to  $\varphi$  (or rate of change of  $\varphi$ ), and, therefore,  $\rho_{\text{NB,T}}(\varphi)$  cannot have sharp enough peaks in the inflationary domain. The only means possible for overcoming this difficulty is, to the best of our knowledge, to go beyond the tree-level approximation. The loop part of the distribution function, depending on the cosmological model, can qualitatively change the predictions of the tree-level theory. We shall show this below for the model with non-minimally coupled instanton accounting for one-loop effects in the quantum ensemble of tunneling Universes.

### 3.1. Hawking-Turok wavefunction and open inflation

Another interesting application of quantum cosmology was the attempt to generate open inflation from Euclidean quantum gravity similarly to the case of the closed Universe. Hawking and Turok have recently suggested the mechanism of quantum creation of an open Universe from the no-boundary cosmological state [29]. Motivated by the observational evidence for the potential possibility of  $\Omega < 1$  and the necessity to avoid a rather contrived nature of inflaton potentials in the early suggestions for open inflation [30, 31] (see also [32]) they constructed a special gravitational instanton. Within the framework of the no-boundary cosmological wavefunction this instanton is capable of generating expanding open homogeneous universes without assuming any special form for the potential. The prior quantum probability of such universes weighted by the anthropic probability of galaxy formation was shown to be peaked at  $\Omega \sim 0.01$ .

Very briefly, the construction of the Hawking-Turok instanton generating the open inflation, as compared to the closed one, is as follows. The inflating Lorentzian spacetime originates in both closed and open models by the nucleation from a 3-dimensional section of the gravitational instanton. In the closed model this is the equatorial section – the boundary of the 4-dimensional quasi-hemisphere labelled by the constant value of the latitude angular coordinate. The analytic continuation of this coordinate into the complex plane gives rise to the Lorentzian quasi-DeSitter spacetime modelling the open inflation. In the open case the Hawking-Turok suggestion was to continue the Euclidean solution beyond the equatorial section up to the point where the Euclidean scale factor again vanishes at the point antipodal to the regular pole on the first hemisphere. The nucleation surface then has to be chosen as the longitudinal section of this quasi-spherical manifold passing through the regular pole

and its antipodal point. Then the analytic continuation of the corresponding longitudinal angle into the complex plane gives rise to the Lorentzian spacetime. The light cone originating from the regular pole cuts in this spacetime the domain sliced by the open spatially homogeneous sections of constant negative curvature. Their chronological succession serves as the model for the open inflationary Universe.

This idea, despite its extremely attractive nature, was criticized from various sides. The Hawking-Turok instanton turned out to be singular, and its singularity raised a number of objections. They were based on the possible instability of flat space [33], originated from the Euclidean theory [34, 35, 36, 37] and from the viewpoint of the resulting timelike singularity in the expanding Universe [35, 38, 37]. The criticism of singular instantons was followed by attempts of their justification by a special treatment of spacetime boundary near the singularity [39] and by considering them as a dimensional reduction from instantons in five dimensions [40] and in 11-dimensional supergravity [41, 42]. Despite the pessimistic conclusions of [41] on the impossibility of inflation in supergravity induced model, it is still very likely that the Hawking-Turok instantons should not be completely ruled out, because their singularity probes Planckian scales where classical equations fail and strong quantum effects might regulate arising infinities. At least, their issue can be regarded open as long as the Hawking-Turok mechanism promises interesting predictions.

The latter was, maybe, the main reason of descending interest in the whole idea, because the practical goal of quantum cosmology – generating the open Universe with observationally justified modern value of  $\Omega$ , not very close to one or zero, – has not been reached. Moreover, the open inflation mechanism essentially used anthropic principle. This, as recognized by the authors of [29], is certainly a retreat in theory, because it makes one to give up on the goal of explaining all the properties of the Universe by using some to constrain others. Another difficulty in the theory of quantum origin of modern Universe arises when it can undergo the self-reproducing inflation scenario [9]. According to [10], this scenario washes away in the observable cosmological data any imprint of quantum initial conditions and, thus, makes their setting meaningless.

In the discussion of these difficulties Linde suggested to replace the no-boundary framework of the open inflation by the approach of tunneling quantum state of the Universe [10]. Such a replacement, according to the discussion above, shifts the probability to larger values of the Hubble constant  $H(\phi) \sim V^{1/2}(\phi)$  and eventually increases the amount of inflation and the value of  $\Omega$ . Then, provided that there is an upper bound on the inflaton or its potential beyond which inflation is impossible, one can get at the point of this bound a particular value of  $\Omega$  fitting into a needed observational range.

This idea was realized by Linde in [10] for two classes of models – nonminimally coupled inflaton field with the *positive* coupling constant  $\xi$  in the interaction term  $-\xi\varphi^2 R/2$  [43, 31] and the model with supergravity induced exponential potentials [44]. In these models the boundary of the inflation domain naturally followed from the positivity of the effective gravitational constant (overall coefficient of the curvature scalar, see next section for details),  $m_P^2/16\pi - \xi\varphi^2/2 > 0$ , or the steepness restrictions on the inflaton potential and could generate a limited amount of inflation with a needed value of  $\Omega$ . The latter property could guarantee the absence of conditions necessary for self-reproducing inflation scenario and, thus, protect the model from washing out the initial conditions in this scenario [10].

However, a simple qualitative analysis of the above type has a number of limitations. First

of all, in the model with nonminimal coupling the instanton action, even in the lowest order of the slow roll expansion, is given by a more complicated expression than (3.7) (see eq. (4.14) below in case of the quartic potential  $\lambda\varphi^4/4$ ). This becomes especially important in case of a big *negative* nonminimal coupling constant,  $\xi < 0$ , which is often regarded preferable from the viewpoint of the CMBR anisotropy [45, 46], because it solves the problem of excessively small  $\lambda$  for the minimal inflaton [9] (it allows one to trade a very small  $\lambda$  in favour of a big  $|\xi| \gg 1$  in the expression for  $\Delta T/T \sim \sqrt{\lambda}/|\xi| \sim 10^{-5}$  in this model). Thus, the expression (3.7) cannot be directly used in the nonminimal model. Secondly, the slow roll corrections should be taken into account, and they become particularly important on the Hawking-Turok instanton in view of its singularity. Finally, tunneling processes in both models of [10] (nonminimal and supergravity induced) start at Planckian energies much beyond reliable perturbative domain, where conventional semiclassical methods are not applicable. This means, that at least lowest order loop effects should be considered and serious arguments found for the justification of loop expansion. Below we show, that with the inclusion of loop effects the mechanism of creation of the inflationary Universe is also possible in the open case, and, as in the closed model, this mechanism belongs to the low-energy domain, which justifies the semiclassical methods.

## 4. Quantum origin of the Universe as a low-energy phenomenon

One can try resolving the difficulties of the above type by resorting to two possibilities: i) changing the Lagrangian of the inflationary model and ii) by going beyond the tree-level approximation. Although the model (3.1) captures all essential features of chaotic inflation theory, there exists important low-energy modification (that is the modification that does not involve curvature-squared and of higher powers in curvatures terms essential only for Planckian scales) that can qualitatively change the above conclusions. This is the model with the inflaton field non-minimally coupled to curvature. Below we consider this model which, together with the inclusion of quantum gravitational loop effects, renders the mechanism of the low-energy phenomenon of the quantum origin of the inflationary Universe compatible with observations.

### 4.1. Non-minimal inflaton coupling

Inflation with non-minimally coupled inflaton is described by the Lagrangian with the inflaton-graviton sector

$$\mathcal{L}(g_{\mu\nu}, \varphi) = \frac{m_P^2}{16\pi} R(g_{\mu\nu}) - \frac{1}{2}\xi\varphi^2 R(g_{\mu\nu}) - \frac{1}{2}(\nabla\varphi)^2 - V(\varphi). \quad (4.1)$$

Such a model has a good family of inflationary solutions for a big negative constant of non-minimal coupling constant  $-\xi = |\xi| \gg 1$ .

This model is of a particular interest for a number of reasons. Firstly, from the phenomenological viewpoint a strong nonminimal coupling allows one to solve the problem of exceedingly small  $\lambda$ . Here the observable magnitude of CMBR anisotropy  $\Delta T/T \sim 10^{-5}$

is proportional to the ratio  $\sqrt{\lambda}/|\xi|$  [45, 46]), so that instead of exceedingly small value of  $\lambda \sim 10^{-13}$  (unacceptable from the viewpoint of reheating theory) one can take big  $|\xi|$  to satisfy the observational constraint. Secondly, this coupling is inevitable from the viewpoint of renormalization theory. Also, among recent implications, it might be important in the theory of an accelerating Universe [47].

Another advantage of this model is the fact that, in the slow-roll regime, it is qualitatively equivalent to the minimal model but with the effective,  $\varphi$ -dependent, Planck mass constant – the overall coefficient of the curvature square

$$m_P^2 \rightarrow m_{\text{eff}}^2(\varphi) = m_P^2 + 8\pi|\xi|\varphi^2. \quad (4.2)$$

For large  $|\xi|\varphi^2$  this constant is much bigger than the original Planck mass, therefore it strongly improves the perturbation expansion in graviton loops, because this expansion runs in inverse powers of  $m_{\text{eff}}^2(\varphi)$ .

Important modification of the distribution function in quantum cosmology of this model occurs already at the tree-level approximation. Now the Euclidean instanton action takes the form

$$I(\varphi) \simeq -\frac{3m_{\text{eff}}^4(\varphi)}{8V(\varphi)}, \quad (4.3)$$

and in view of nontrivial  $\varphi$ -dependence of  $m_{\text{eff}}^4(\varphi)$  the shape of the distribution function can acquire new interesting probability peaks. Interestingly, for a particular inflaton potential this model actually suggest a natural resolution of the problem mentioned in Introduction – the wiping out of initial conditions for inflation due to exponentially growing anthropic factor. Indeed, if one takes the simplest quadratic potential without higher (quartic) terms, then as one can check it will correspond to approximately constant,  $\varphi$ -independent, value of the Hubble constant,

$$V(\varphi) = \frac{1}{2}m^2\varphi^2, \quad H^2 \sim \frac{m^2}{6|\xi|} = \text{const}, \quad (4.4)$$

and give rise to the total probability – the fundamental *tunneling* probability factor times the anthropic factor

$$\rho_T^{\text{total}}(\varphi) \sim \exp \left( -\frac{3m_P^4}{4m^2\varphi^2} - \frac{48\pi^2\xi^2\varphi^2}{m^2} + \frac{3mt}{\sqrt{6|\xi|}} \right). \quad (4.5)$$

This distribution has a strong damping for both large and small  $\varphi$ , while the anthropic factor at all turns out to be  $\varphi$ -independent. Therefore, the probability peak at  $\varphi \sim m_P/\sqrt{|\xi|}$  never gets washed away due to anthropic aspects of self-reproducing inflation.

Unfortunately this theory can hardly be considered seriously, because the restriction by simplest quadratic potential – the crucial property guaranteeing the above quasi-gaussian in  $\varphi$  behaviour – is not consistent from the viewpoint of quantum theory. Renormalization effects would always generate quartic and other terms in the inflaton potential which would immediately destroy this nice picture. So let us consider another source of possible mechanism for initial conditions for inflation – quantum loop contributions to the distribution function.

## 4.2. Beyond tree level – one-loop effect of inhomogeneous modes

Beyond the tree level the distribution function (2.11) – the diagonal element of the reduced density matrix (2.11) – is no longer given by just the square of the amplitude of the wavefunction. Now the preexponential factor starts playing important role and, in addition, nontrivial factor is being contributed due to the integration over microscopic modes  $f^T$ . For no-boundary and tunneling states these new contributions essentially modify the tree-level algorithm. Interestingly, the expressions (3.5) and (3.6) get replaced by

$$\rho_{\text{NB,T}}(\varphi) \sim \exp[\mp I(\varphi) - \mathbf{\Gamma}(\varphi)], \quad (4.6)$$

where the classical Euclidean action  $I(\varphi)$  on the quasi-DeSitter instanton with the inflaton field  $\varphi$  – the 4-dimensional sphere of the radius inverse to the Hubble constant  $H(\varphi)$  – is amended by the loop effective action  $\mathbf{\Gamma}(\varphi)$  calculated on the same instanton [48, 49, 16, 53]. This action begins with the one-loop functional determinant of the inverse propagator of the full set of quantum fields  $\phi$

$$\mathbf{\Gamma} = \frac{1}{2} \text{Tr} \ln \frac{\delta^2 \mathbf{I}[\phi]}{\delta \phi \delta \phi} + \dots \quad (4.7)$$

The one-loop contribution can qualitatively change predictions of the tree-level theory due to the dominant part of the effective action induced by the anomalous scaling behaviour. On the instanton of small size  $1/H(\varphi)$  this asymptotic behaviour looks like

$$\mathbf{\Gamma}(\varphi) \sim Z \ln H(\varphi), \quad (4.8)$$

where  $Z$  is the total anomalous scaling of all quantum fields in the model. This quantity is directly related to the conformal anomaly of the theory integrated over the instanton volume. For heavy fields it is dominated by the sum of terms quartic in their mass parameters.

In the non-minimal inflaton model of [50, 51] the inflaton potential is taken to contain the quartic term

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2 + \frac{\lambda}{4} \varphi^4. \quad (4.9)$$

Also, it is natural to assume that this model contains generic GUT sector of Higgs  $\chi$ , vector gauge  $A_\mu$  and spinor fields  $\psi$  coupled to the inflaton via the interaction term

$$\mathbf{L}_{\text{int}} = \sum_{\chi} \frac{\lambda_{\chi}}{4} \chi^2 \varphi^2 + \sum_A \frac{1}{2} g_A^2 A_\mu^2 \varphi^2 + \sum_{\psi} f_{\psi} \varphi \bar{\psi} \psi + \text{derivative coupling}. \quad (4.10)$$

For such a model the parameter  $Z$  can be very big, because of the Higgs effect generating large masses of all the particles directly coupled to the inflaton. Due to this effect the anomalous scaling (dominated by terms quartic in particle masses) is quadratic in  $|\xi|$ ,

$$Z = 6 \frac{|\xi|^2}{\lambda} \mathbf{A}, \quad (4.11)$$

$$\mathbf{A} = \frac{1}{2\lambda} \left( \sum_{\chi} \lambda_{\chi}^2 + 16 \sum_A g_A^4 - 16 \sum_{\psi} f_{\psi}^4 \right), \quad (4.12)$$

with a particular coefficient  $\mathbf{A}$  – a universal combination of the coupling constants above.

Thus, the probability peak in this model reduces to the extremum of the function

$$\ln \rho_{\text{NB, T}}(\varphi) \simeq \mp I(\varphi) - 3 \frac{|\xi|^2}{\lambda} \mathbf{A} \ln \frac{\varphi^2}{\mu^2}. \quad (4.13)$$

in which the  $\varphi$ -dependent part of the classical instanton (4.3) action is very different from its analogue in the minimal model (3.7). When expanded in powers of the slow roll expansion parameter,  $m_P^2/|\xi|\varphi^2 \ll 1$ , this action equals

$$I(\varphi) = -\frac{96\pi^2|\xi|^2}{\lambda} - \frac{24\pi(1+\delta)|\xi|}{\lambda} \frac{m_P^2}{\varphi^2} + O\left(\frac{m_P^4}{\varphi^4}\right), \quad (4.14)$$

$$\delta \equiv -\frac{8\pi|\xi|m^2}{\lambda m_P^2}. \quad (4.15)$$

Thus the analysis of probability maxima does not reduce to considering the extrema of the potential. Rather, at the probability maximum the contribution of the instanton action gets balanced by the anomalous scaling term, provided the signs of  $(1+\delta)$  and  $\mathbf{A}$  are properly correlated with the  $(\mp)$  signs of the no-boundary (tunneling) proposals. As a result the probability peak exists with parameters – mean values of the inflaton and Hubble constants and relative width

$$\varphi_I^2 = m_P^2 \frac{8\pi|1+\delta|}{|\xi|\mathbf{A}}, \quad (4.16)$$

$$H^2(\varphi_I) = m_P^2 \frac{\lambda}{|\xi|^2} \frac{2\pi|1+\delta|}{3\mathbf{A}}, \quad (4.17)$$

$$\frac{\Delta\varphi}{\varphi_I} \sim \frac{\Delta H}{H} \sim \frac{1}{\sqrt{12\mathbf{A}}} \frac{\sqrt{\lambda}}{|\xi|}, \quad (4.18)$$

which are strongly suppressed by a small ratio  $\sqrt{\lambda}/|\xi|$  known from the COBE normalization for  $\Delta T/T \sim 10^{-5}$  [54, 55] (because the CMBR anisotropy in this model is proportional to this ratio [45, 46]). This GUT scale peak gives rise to the inflationary epoch with the e-folding number

$$N \simeq \frac{48\pi^2}{\mathbf{A}} \quad (4.19)$$

only for  $1+\delta > 0$  and, therefore, only for the *tunneling* quantum state (plus sign in (4.13)). Comparison with  $N \geq 60$  necessary for  $\Omega > 1$  immediately yields the bound on  $\mathbf{A}$  [52],

$$\mathbf{A} \sim 5.5, \quad (4.20)$$

which can be regarded as a selection criterion for particle physics models [50, 51]. These conclusions on the nature of the inflation dynamics from the initial probability peak remain true also at the quantum level – with the effective equations replacing the classical equations of motion [52].

### 4.3. Open inflation without anthropic principle

It is interesting that the synthesis of the Hawking-Turok paradigm with the model of a strong non-minimal coupling and a proper account of loop corrections gives a two-fold result: it does not only generate a low-energy mechanism of creation of the open Universe with a resonable value of  $\Omega$  (not very close to zero or one) but also allows one to abandon the anthropic principle inherent in the original model of [29]. This goes as follows [56].

The tree-level Euclidean action of the Hawking-Turok instanton in the non-minimal model can be approximately calculated as an expansion of the slow-roll parameter  $m_P^2/|\xi|\varphi^2$ , similarly to the analogous expansion for the Hartle-Hawking instanton (4.14). In contrast with (4.14), however, this expansion has to be performed up to the second order in  $m_P^2/|\xi|\varphi^2$  inclusive (first order approximation turns out to be insufficient as shown in [56]) and it reads

$$I_{HT}(\varphi) = -\frac{96\pi^2|\xi|^2}{\lambda} - \frac{2\pi(1+\delta)}{\lambda} \frac{m_P^2}{\varphi^2} + 2\frac{(1+\delta)^2}{\lambda} \left(\frac{m_P^2}{\varphi^2}\right)^2 \ln\left(\frac{6\pi|\xi|\varphi^2}{m_P^2(1+\delta)\kappa}\right) + O\left(\frac{m_P^6}{|\xi|\varphi^6}\right), \quad (4.21)$$

where  $\kappa$  absorbs the combination of numerical parameters  $\ln \kappa \equiv 11/3 - \ln 4 - 3(1+2\delta)^2/4(1+\delta)^2$  (apart from the negligible dependence of  $\kappa$  on  $\delta$ ,  $\kappa \sim 4.6$ ). Due to big  $|\xi|$  it contains a large but slowly varying (in  $\varphi$ ) logarithmic term with positive coefficient. It arises as a rather nontrivial contribution of the singular point of the Hawking-Turok instanton and turns out to be crucial for the mechanism of low-energy origin of open inflation.

With this Euclidean action the Hawking-Turok distribution function (in both no-boundary and tunneling incarnations) have nontrivial probability peaks already in the tree-level approximation, but they correspond to the values of  $\Omega$  too close either to one or to zero. With the inclusion of the one-loop effective action the situation qualitatively changes, just like in the closed case. It turns out that the new probability peak arises. For the *no-boundary* proposal, it has the following parameters – the mean value  $\varphi_I$ , Hubble constant  $H_I$  and quantum dispersion:

$$\varphi_I^2 \simeq \frac{m_P^2}{|\xi|} \left( \frac{2}{3\mathbf{A}} \ln \frac{24\pi^2}{\kappa^2 e \mathbf{A}} \right)^{1/2}, \quad (4.22)$$

$$H_I^2 \simeq m_P^2 \frac{\lambda}{|\xi|^2} \frac{1}{12} \left( \frac{2}{3\mathbf{A}} \ln \frac{24\pi^2}{\kappa^2 e \mathbf{A}} \right)^{1/2}, \quad (4.23)$$

$$\frac{\Delta\varphi}{\varphi_I} \sim \frac{\Delta H}{H_I} \sim \frac{\kappa^2 \sqrt{6\mathbf{A}}}{72\pi^2} \frac{\sqrt{\lambda}}{|\xi|}. \quad (4.24)$$

Similarly to the closed model, these parameters are suppressed relative to the Planck scale by a small dimensionless ratio  $\sqrt{\lambda}/|\xi|$  known from the COBE normalization. As regards the e-folding number  $N$ , it is given for this peak entirely in terms of the same universal combination of coupling constants (4.12)

$$N \simeq \left( \frac{24\pi^2}{\mathbf{A}} \ln \frac{24\pi^2}{\kappa^2 e \mathbf{A}} \right)^{1/2}. \quad (4.25)$$

Comparison of this result with the e-folding number,  $N \sim 60$ , necessary for generating the observable density  $\Omega$ ,  $0 < \Omega < 1$ , not very close to one or zero, immediately gives the bound on  $\mathbf{A}$

$$\mathbf{A} \sim \frac{48\pi^2}{N^2} \ln \frac{N}{\kappa e} \sim 0.3. \quad (4.26)$$

A similar analysis for the case of the *tunneling* distribution function shows that its probability maximum corresponds to the e-folding number,  $N \simeq \kappa\sqrt{e} \sim 8$ , and the density parameter  $\Omega \sim e^{-100}$  which are far too small to describe the observable Universe. This leaves us with the only candidate for the initial conditions of inflation (4.22)-(4.24) generated by the no-boundary Hawking-Turok wavefunction of the open Universe.

## 5. Conclusions

Thus, despite conceptual and technical problems, modern quantum cosmology represents viable theory capable of predictions and fundamental (quantum gravitational) justification of the origin of the inflationary Universe. It seems to be consistently describing the low-energy phenomenon of such origin which matches with the main observable cosmological parameters – CMBR anisotropy, admissible values of the density parameter  $\Omega$ , restrictions on the duration of inflationary epoch, etc. It also belongs the GUT energy scale which is much below the Planckian one and, thus, can be justified within the field-theoretical loop expansion. Unfortunately, these phenomenologically attractive results do not resolve the controversy between the no-boundary and tunneling cosmological states. The tunneling state represents a viable scheme for a closed Universe, while the tunneling one – for an open model within the Hawking-Turok prescription. Both states has the right for existence and, apparently, wait for the proper moment when they will be naturally included in some unifying framework, like third quantization or theory of baby universes – bright ideas existing now, however, only at a very speculative level [58].

From the phenomenological viewpoint, quantum cosmology is expected to be invoked for the explanation of such recently observed phenomena like the acceleration of the Universe [57]. The first attempt to use it as an alternative to quintessence [59] has unfortunately failed [60].

In general, at the turn of new millennium the ideas of quantum cosmology are anticipated to enrich other fields and to be unified with other concepts of cosmological high energy physics and theory of multi-dimensional compactifications induced by superstring theory. For example, the origin of big non-minimal curvature coupling of the inflaton in the model of the above type might be accounted for within the Randall-Sundrum two-branes model [61], which as is known [62] induces nonminimal coupling in the effective 4-dimensional action. On the other hand, the Hamilton-Jacobi equation method intensively used in context of AdS/CFT correspondence and D-brane physics [63] has its origin in canonical quantum gravity and its cosmological implications. It is good to see how old problems are finally springing a leak and new revelations are expecting us.

## Acknowledgements

This work was supported by the Russian Foundation for Basic Research under the grant No 99-02-16122, by the grant of support of leading scientific schools No 00-15-96699 and also by the Russian Research program “Cosmomicrophysics”.

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